(Time: 3 Hours) **Max. Marks: 80**

N.B. (1) Question No. 1 is compulsory.

(2) Answer any three questions from Q.2 to Q.6.

(3) Use of Statistical Tables permitted.

(4) Figures to the right indicate full marks

Q1 a) If $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$, then find the Eigen values of $4A^{-1} + A^3 + I$ [5]

b) Evaluate $\int_c |z|\, dz$, where C is the left half of unit circle $|z| = 1$ from z = - i to z = i. [5]

c) $Maximise \quad z = x_1 + 3x_2 + 3x_3$ [5]

Subject to $\quad x_1 + 2x_2 + 3x_3 = 4$

$2x_1 + 3x_2 + 5x_3 = 7.$

Find all the basic solutions to the above problem. Which of them are basic feasible, non-degenerate, infeasible basic and optimal solution.

d) Tests made on breaking strength of 10 pieces of a metal wire gave the following results

578, 572, 570, 568, 572, 570, 570, 572, 596 and 584 in kgs. [5]

Test if the breaking strength of the metal wire can be assumed to be 577 kg ?

Q2 (a) Using Cauchy's residue theorem evaluate [6]

$\int_C \frac{(z+4)^2}{z^4+5z^3+6z^2} dz,$ Where c is |z|=1.

(b) Find $Z\{f(k) * g(k)\} \quad if \quad f(k) = 4^k\, U(k)\, , g(k) = 5^k U(k).$ [6]

(c) Solve the following L.P.P by Simplex Method [8]

$Maximise \quad z = 3x_1 + 2x_2 + 5x_3$

Subject to $\quad x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 2x_3 \leq 460$

$x_1 + 4x_2 \leq 420$

$x_1, x_2, x_3 \geq 0$

Q3 a) Theory predicts that the proportion of beans in the four groups A, B, C, D should be 9: 3 :3 :1. In an experiment among 1600 beans the numbers in the four groups were 882, 313, 287 and 118. Does the experimental results support the theory? [6]

(Given that Critical value of chi-square 3 d. f and 5% L.O.S is 7.81)

b) Obtain Taylor's and Laurent's series expansion of $f(z) = \frac{z-1}{z^2-2z-3}$ [6]

c) Use the method of Lagrange's multipliers to solve the following N.L.P.P [8]

Optimize $z = 6x_1 + 8x_2 - x_1^2 - x_2^2$

Subject to $4x_1 + 3x_2 = 16$,

$3x_1 + 5x_2 = 15$

$x_1, x_2 \geq 0$

Q4a) fit a Poisson distribution to the following data [6]

No. of deaths	0	1	2	3	4
Frequencies	123	59	14	3	1

b) Find the inverse Z-transform of $\frac{1}{(z-2)(z-3)}$, $if\ ROC\ is\ \ (i)\ \ |z| < 2\ \ (ii)\ 2 < |z| < 3$ [6]

c) Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalizable. Find the transforming matrix and the diagonal matrix. [8]

Q5a) Using the method of Lagrange's multipliers to solve the following N.L.P.P [6]

Optimize $z = 4x_1 + 8x_2 - x_1^2 - x_2^2$

Subject to $x_1 + x_2 = 4$,

$x_1, x_2 \geq 0$. [6]

b) Verify Cayley- Hamilton Theorem for the matrix $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{bmatrix}$ [6]

c) Solve by the dual Simplex Method [8]

$Minimise\ \ z = 6x_1 + x_2$

Subject to $2x_1 + x_2 \geq 3$,

$x_1 - x_2 \geq 0$, $x_1, x_2 \geq 0$

Q6a) Find the Z-transform of $f\{k\} = \begin{cases} b^k, & k < 0 \\ a^k, & k \geq 0 \end{cases}$ [6]

b) The income of a group of 10,000 persons were found to be normally distributed with mean Rs.520 and standard deviation Rs.60. Find the lowest income of the richest 500. [6]

c) Using Kuhn Tucker conditions, solve the following NLPP [8]

$Maximise$ $z = 10x_1 + 4x_2 - 2x_1^2 - x_2^2$

Subject to $2x_1 + x_2 - 5 \leq 0$

$x_1, x_2 \geq 0$
